

1 次の式を展開せよ。

(1) $(a+3)^3$

(2) $(x-2)^3$

(3) $(2x-1)^3$

(4) $(x+4y)^3$

(5) $(3x+2y)^3$

(6) $(-2a+b)^3$

2 次の式を展開せよ。

(1) $(x+4)(x^2-4x+16)$

(2) $(a-2)(a^2+2a+4)$

(3) $(x-4y)(x^2+4xy+16y^2)$

(4) $(3x+5y)(9x^2-15xy+25y^2)$

3 次の式を因数分解せよ。

(1) x^3+8

(2) a^3-125b^3

(3) $8x^3+27$

(4) $64a^3-27b^3$

(5) $\frac{1}{27}+x^3$

(6) $x^3-y^3z^3$

4 次の式を因数分解せよ。

(1) $64x^6-1$

(2) $1-a^6$

5 次の式を展開せよ。

(1) $(x-y)^2(x^2+xy+y^2)^2$

(2) $(a+b+c)^3$

6 次の式を因数分解せよ。

(1) x^3-3x^2+6x-8

(2) $a^3-9a^2+27a-27$

1 (1) (与式) $= a^3 + 3 \cdot a^2 \cdot 3 + 3 \cdot a \cdot 3^2 + 3^3 = a^3 + 9a^2 + 27a + 27$

(2) (与式) $= x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3 = x^3 - 6x^2 + 12x - 8$

(3) (与式) $= (2x)^3 - 3 \cdot (2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 - 1^3 = 8x^3 - 12x^2 + 6x - 1$

(4) (与式) $= x^3 + 3 \cdot x^2 \cdot 4y + 3 \cdot x \cdot (4y)^2 + (4y)^3 = x^3 + 12x^2y + 48xy^2 + 64y^3$

(5) (与式) $= (3x)^3 + 3 \cdot (3x)^2 \cdot 2y + 3 \cdot 3x \cdot (2y)^2 + (2y)^3 = 27x^3 + 54x^2y + 36xy^2 + 8y^3$

(6) (与式) $= (-2a)^3 + 3 \cdot (-2a)^2 \cdot b + 3 \cdot (-2a) \cdot b^2 + b^3 = -8a^3 + 12a^2b - 6ab^2 + b^3$

2 (1) (与式) $= (x+4)(x^2 - x \cdot 4 + 4^2) = x^3 + 4^3 = x^3 + 64$

(2) (与式) $= (a-2)(a^2 + a \cdot 2 + 2^2) = a^3 - 2^3 = a^3 - 8$

(3) (与式) $= (x-4y)\{x^2 + x \cdot 4y + (4y)^2\} = x^3 - (4y)^3 = x^3 - 64y^3$

(4) (与式) $= (3x+5y)\{(3x)^2 - 3x \cdot 5y + (5y)^2\} = (3x)^3 + (5y)^3 = 27x^3 + 125y^3$

3 (1) (与式) $= x^3 + 2^3 = (x+2)(x^2 - x \cdot 2 + 2^2) = (x+2)(x^2 - 2x + 4)$

(2) (与式) $= a^3 - (5b)^3 = (a-5b)\{a^2 + a \cdot 5b + (5b)^2\} = (a-5b)(a^2 + 5ab + 25b^2)$

(3) (与式) $= (2x)^3 + 3^3 = (2x+3)\{(2x)^2 - 2x \cdot 3 + 3^2\} = (2x+3)(4x^2 - 6x + 9)$

(4) (与式) $= (4a)^3 - (3b)^3 = (4a-3b)\{(4a)^2 + 4a \cdot 3b + (3b)^2\} = (4a-3b)(16a^2 + 12ab + 9b^2)$

(5) (与式) $= \left(\frac{1}{3}\right)^3 + x^3 = \left(\frac{1}{3} + x\right)\left\{\left(\frac{1}{3}\right)^2 - \frac{1}{3} \cdot x + x^2\right\} = \left(\frac{1}{3} + x\right)\left(\frac{1}{9} - \frac{1}{3}x + x^2\right)$

(6) (与式) $= x^3 - (yz)^3 = (x-yz)\{x^2 + x \cdot yz + (yz)^2\} = (x-yz)(x^2 + xyz + y^2z^2)$

4 (1) (与式) $= (8x^3)^2 - 1^2 = (8x^3+1)(8x^3-1)$

$$= (2x+1)(4x^2-2x+1)(2x-1)(4x^2+2x+1)$$

$$= (2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$$

別解 (与式) $= (4x^2)^3 - 1^3 = (4x^2-1)(16x^4+4x^2+1)$

$$= (4x^2-1)\{(4x^2+1)^2 - 4x^2\}$$

$$= (2x+1)(2x-1)(4x^2+1-2x)(4x^2+1+2x)$$

$$= (2x+1)(2x-1)(4x^2-2x+1)(4x^2+2x+1)$$

(2) (与式) $= 1^2 - (a^3)^2 = (1+a^3)(1-a^3)$

$$= (1+a)(1-a+a^2)(1-a)(1+a+a^2)$$

$$= (1+a)(1-a)(1-a+a^2)(1+a+a^2)$$

別解 (与式) $= 1^3 - (a^2)^3 = (1-a^2)(1+a^2+a^4)$

$$= (1-a^2)\{(1+a^2)^2 - a^2\}$$

$$= (1+a)(1-a)(1+a^2-a)(1+a^2+a)$$

$$= (1+a)(1-a)(1-a+a^2)(1+a+a^2)$$

$$\begin{aligned}
 \boxed{5} \quad (1) \quad (\text{与式}) &= \{(x-y)(x^2+xy+y^2)\}^2 \\
 &= (x^3-y^3)^2 \\
 &= (x^3)^2 - 2x^3y^3 + (y^3)^2 \\
 &= x^6 - 2x^3y^3 + y^6
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad (\text{与式}) &= \{(a+b)+c\}^3 \\
 &= (a+b)^3 + 3(a+b)^2c + 3(a+b)c^2 + c^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3 + 3(a^2+2ab+b^2)c + 3ac^2 + 3bc^2 + c^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3 \\
 &= a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3b^2c + 3bc^2 + 3c^2a + 3ca^2 + 6abc
 \end{aligned}$$

$$\begin{aligned}
 \boxed{6} \quad (1) \quad (\text{与式}) &= (x^3-8) - (3x^2-6x) \\
 &= (x^3-2^3) - 3x(x-2) \\
 &= (x-2)(x^2+2x+4) - 3x(x-2) \\
 &= (x-2)\{(x^2+2x+4) - 3x\} \\
 &= (x-2)(x^2-x+4)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad (\text{与式}) &= (a^3-27) - (9a^2-27a) \\
 &= (a^3-3^3) - 9a(a-3) \\
 &= (a-3)(a^2+3a+9) - 9a(a-3) \\
 &= (a-3)(a^2+3a+9-9a) \\
 &= (a-3)(a^2-6a+9) \\
 &= (a-3)^3
 \end{aligned}$$

別解 (展開の公式 $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ を逆に利用する)

$$(\text{与式}) = a^3 - 3 \cdot a^2 \cdot 3 + 3 \cdot a \cdot 3^2 - 3^3 = (a-3)^3$$